I. Solution by H. M. ARMSTRONG, Cooch's Bridge, Delaware.

Let the sides of the given triangle be denoted by the vectors 2α, 2β, and $2(\alpha+\beta)$. Let i denote a unit vector perpendicular to the plane of the triangle. Then

$$\rho_1=\alpha+\frac{1}{\sqrt{3}}i\alpha \text{.........}(1),\quad \rho_2=2\alpha+\beta+\frac{1}{\sqrt{3}}ei\beta\text{.........}(2),\quad \rho_3=\alpha+\beta-ei(\alpha+\beta)\text{.........}(3)$$

are the vectors drawn to the vertices of the three isosceles triangles.

Hence the vectors of the sides of the new triangle are obviously $\rho_2-\rho_1$, $\rho_3-\rho_2$, $\rho_1-\rho_2$. By means of (1), (2), and (3), we find that $(\rho_2-\rho_1)^2=(\rho_3-\rho_2)^2=(\rho_1-\rho_2)^2$.

Hence $T(\rho_2-\rho_1)=T(\rho_3-\rho_2)=T(\rho_1-\rho_2)$. Therefore the triangle is equilateral.

II. Solution by G. W. GREENWOOD, M. A. (Oxon). Lebanon, Ill.

Denote the vertices by A, B, C, and the remaining vertices of the triangles adjacent to BC, CA, AB, by A', B', C', respectively. I will assume that the triangles are all exterior to the given triangle. Then

$$A'C=a/\sqrt{3},\quad B'C=b/\sqrt{3},\quad \angle A'CB'=60^\circ+C.$$
$$\begin{aligned} A'B'^2&=\tfrac{1}{3}[a^2+b^2-2ab\cos(60+C)]\\ &=\tfrac{1}{3}[a^2+b^2-ab(\cos C-\sqrt{3}\sin C)]\\ &=\tfrac{1}{6}[a^2+b^2+c^2+2\sqrt{3}\ ab\sin C]\\ &=\tfrac{1}{6}[a^2+b^2+c^2+\sqrt{3}\ abc/r],\end{aligned}$$

where r is the radius of the circumcircle of the triangle ABC. The result shows that $A'B'=B'C'=C'A'$. The same result will follow if A' and A are on the same side of BC, etc.

Also solved by M. E. Graber, J. Scheffer, and G. B. M. Zerr. G. I. Hopkins refers to solution of Problem 96, Vol. 5, No. 4.

245. Proposed by J. H. M. MACLAGAN-WEDDERBURN, M. A., Chicago, Ill.

Given two lines of fixed length, $AB=a$, $BC=b$, perpendicular to each other. A line CP is drawn making $\angle BPC=\theta$. AD is the perpendicular to AB meeting CP in D. Find by Euclidean construction the angle θ such that $AD^2\cos^2\theta+b^2\sin^2\theta$ is a minimum.

Solution by the PROPOSER.

Draw any line CP. From B drop the perpendicular BL upon CP; draw AM parallel to PC meeting BL in M. Join CM. Then $CM^2=CL^2+LM^2=b^2\sin^2\theta+AD^2\cos^2\theta$. The locus of M is a circle described on AB as a diameter with O, the middle point of AB, as center. In order that CM^2, and therefore CM shall be a minimum, the line CM must pass through O.

The construction is therefore the following: Bisect AB in O; produce BOA to P so that $OP=OC$. Then BPC=required angle θ. [Newton's *Principia*, Vol. 2, Section 7, Proposition XXXIV, Scholium.]